Beyond Content:
Growing Matheamtical Thinkers as a Collaborative Team

Oklahoma City Public Schools
June 2, 2015

Solution Tree

A Solution Tree Event

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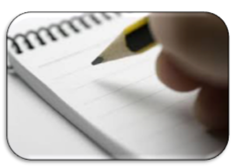
Throughout the session…

Keep a list of strategies used or referenced in the session.



Why do we need the Process Standards?

What is quick (30 second) message to:

- Administrators?
- Teachers?
- Parents?
- Students?

Track Your Progress

Beyond Content: Growing Mathematical Thinkers as a Collaborative Team

Shade each rectangle to show your current understanding of each learning target.

- I can incorporate strategies to teach math that engage students in learning through the process standards.

Starting …	Getting There …	Got It!

- I can create quality lessons that include both the content standards and the process standards.

Starting …	Getting There …	Got It!

Strategies:

Vision:

Geometry Process Priority Academic Student Skills

Process Standard 1: Problem Solving	
HS.1.1	Apply a wide variety of problem-solving strategies (identify a pattern, use equivalent representations) to solve problems from within and outside mathematics.
HS.1.2	Identify the problem from a described situation, determine the necessary data and apply appropriate problem-solving strategies.
Process Standard 2: Communication	
HS.2.1	Use mathematical language and symbols to read and write mathematics and to converse with others.
HS.2.2	Demonstrate mathematical ideas orally and in writing.
HS.2.3	Analyze mathematical definitions and discover generalizations through investigations.
Process Standard 3: Reasoning	
HS.3.1	Use various types of logical reasoning in mathematical contexts and real-world situations.
HS.3.2	Prepare and evaluate suppositions and arguments.
HS.3.3	Verify conclusions, identify counterexamples, test conjectures, and justify solutions to mathematical problems.
HS.3.4	Justify mathematical statements through proofs.
Process Standard 4: Connections	
HS.4.1	Link mathematical ideas to the real world (e.g., statistics helps qualify the confidence we can have when drawing conclusions based on a sample).
HS.4.2	Apply mathematical problem-solving skills to other disciplines.
HS.4.3	Use mathematics to solve problems encountered in daily life.
HS.4.4	Relate one area of mathematics to another and to the integrated whole (e.g., connect equivalent representations to corresponding problem situations or mathematical concepts).
Process Standard 5: Representation	
HS.5.1	Use algebraic, graphic, and numeric representations to model and interpret mathematical and real world situations.
HS.5.2	Use a variety of mathematical representations as tools for organizing, recording, and communicating mathematical ideas (e.g., mathematical models, tables, graphs, spreadsheets).
HS.5.3	Develop a variety of mathematical representations that can be used flexibly and appropriately.

8th Grade Math Process Priority Academic Student Skills

Process Standard 1: Problem Solving	
MS.1.1	Develop and test strategies to solve practical, everyday problems which may have single or multiple answers.
MS.1.2	Use technology to generate and analyze data to solve problems.
MS.1.3	Formulate problems from situations within and outside of mathematics and generalize solutions and strategies to new problem situations.
MS.1.4	Evaluate results to determine their reasonableness.
MS.1.5	Apply a variety of strategies (e.g., restate the problem, look for a pattern, diagrams, solve a simpler problem, work backwards, trial and error) to solve problems, with emphasis on multistep and non-routine problems.
MS.1.6	Use oral, written, concrete, pictorial, graphical, and/or algebraic methods to model mathematical situations.
Process Standard 2: Communication	
MS.2.1	Discuss, interpret, translate (from one to another) and evaluate mathematical ideas (e.g., oral, written, pictorial, concrete, graphical, algebraic).
MS.2.2	Reflect on and justify reasoning in mathematical problem solving (e.g., convince, demonstrate, formulate).
MS.2.3	Select and use appropriate terminology when discussing mathematical concepts and ideas.
Process Standard 3: Reasoning	
MS.3.1	Identify and extend patterns and use experiences and observations to make suppositions.
MS.3.2	Use counter examples to disprove suppositions (e.g., all squares are rectangles, but are all rectangles squares?).
MS.3.3	Develop and evaluate mathematical arguments (e.g., agree or disagree with the reasoning of other classmates and explain why).
MS.3.4	Select and use various types of reasoning (e.g., recursive [loops], inductive [specific to general], deductive [general to specific], spatial, and proportional).
Process Standard 4: Connections	
MS.4.1	Apply mathematical strategies to solve problems that arise from other disciplines and the real world.
MS.4.2	Connect one area or idea of mathematics to another (e.g., relates equivalent number representations to each other, relate experiences with geometric shapes to understanding ratio and proportion).
Process Standard 5: Representation	
MS.5.1	Use a variety of representations to organize and record data (e.g., use concrete, pictorial, and symbolic representations).
MS.5.2	Use representations to promote the communication of mathematical ideas (e.g., number lines, rectangular coordinate systems, scales to illustrate the balance of equations).
MS.5.3	Develop a variety of mathematical representations that can be used flexibly and appropriately (e.g., base-10 blocks to represent fractions and decimals, appropriate graphs to represent data).
MS.5.4	Use a variety of representations to model and solve physical, social, and mathematical problems (e.g., geometric objects, pictures, charts, tables, graphs).

[Illustrative Mathematics]

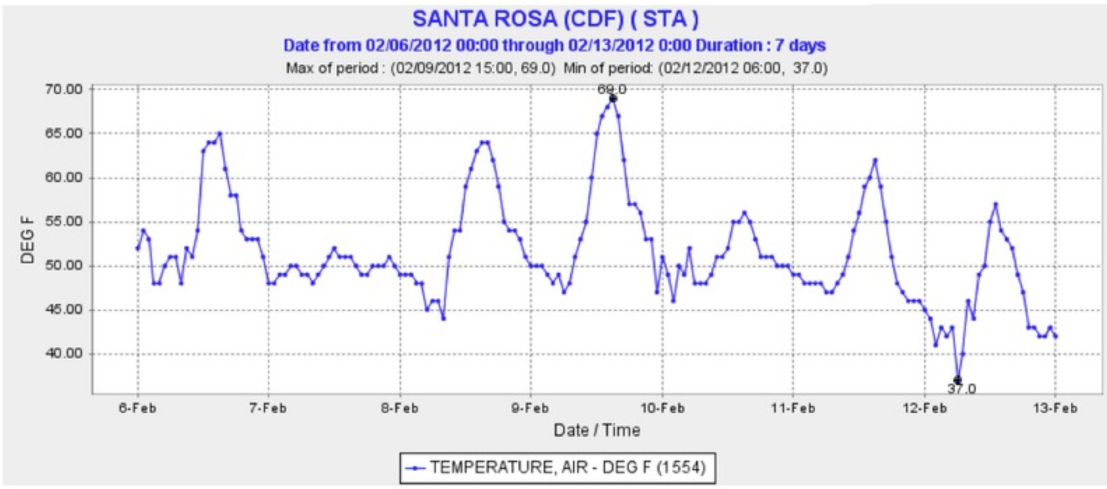
F-IF Telling a Story With Graphs

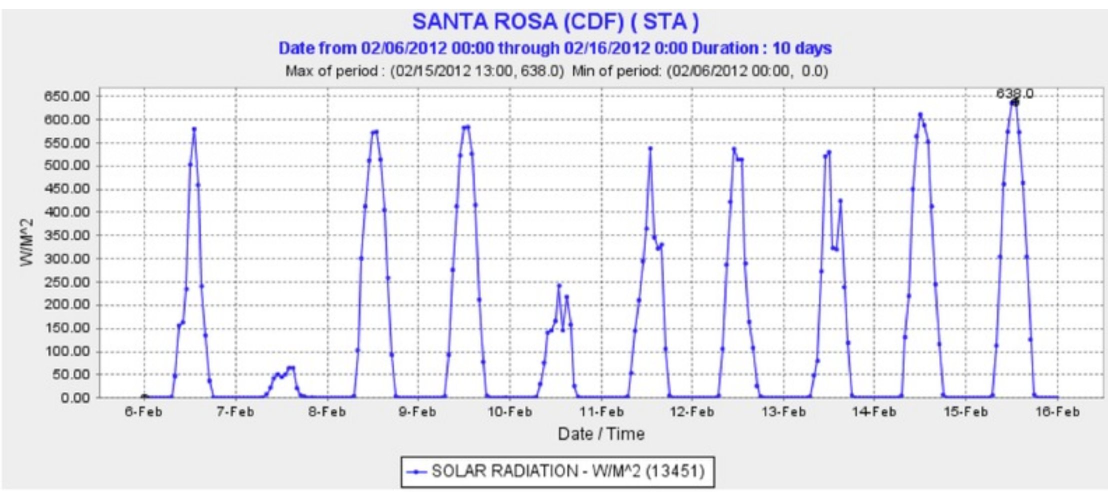
[Alignment 1: F-IF.B.4]

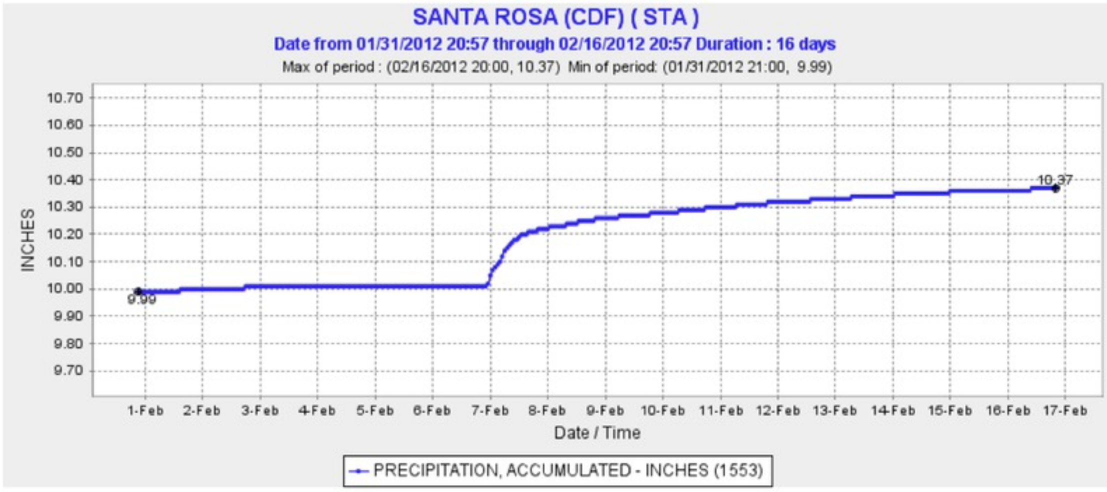
Each of the following graphs tells a story about some aspect of the weather: temperature (in degrees Fahrenheit), solar radiation (in watts per square meters), and cumulative rainfall (in inches)) measured by sensors in Santa Rosa, CA in February 2012. Note that the vertical gridlines represent the start of the day whose date is given.

a. Give a verbal description of the function represented in each graph. What does each function tell you about the weather in Santa Rosa?

b. Tell a more detailed story using information across several graphs. What are the connections between the graphs?






SANTA ROSA (CDF) (STA)
Date from 01/31/2012 20:57 through 02/16/2012 20:57 Duration : 16 days
Max of period : (02/16/2012 20:00, 10.37) Min of period: (01/31/2012 21:00, 9.99)
INCHES
Date / Time
PRECIPITATION, ACCUMULATED - INCHES (1553)

My Math Class

What does a typical math class look like? How does it begin? What happens in the middle? How does it end?	**What are the *students* doing?**	**What is the *teacher* doing?**

Shifts in Teaching and Learning

6 – 8	9 – 12
$\frac{5}{12} = \frac{x}{30}$	$x^2 + 5x + 4 = 0$

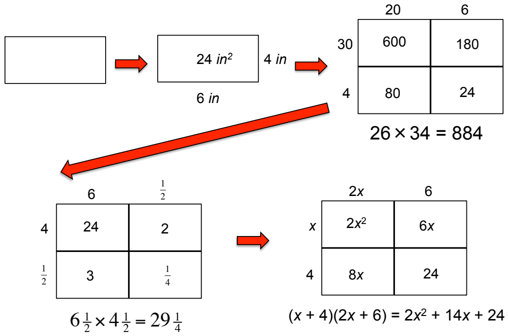
From algorithms:

Solve one of the problems above using an algorithm.

To multiple representations/strategies:

Work as a collaborative team to explain two different strategies you can use to simplify the expression or solve the equation that show your understanding of the problem.

Connections

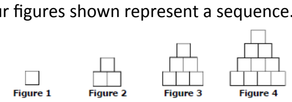


3 E's

Is the model/representation

- **E**asy to use?
- **E**ffective to use?
- **E**fficient to use?

The four figures shown represent a sequence.



Enter your answer for Figure 5 in the first response box.

Figure	Number of Squares
1	1
2	3
3	6
4	10
5	?

Figure 40 in this sequence contains 820 squares. How many squares are needed to make Figure 41? Enter your answer for Figure 41 in the second response box.

--Smarter Balanced Assessment Consortium Grade 5

Always, Sometimes, or Never?

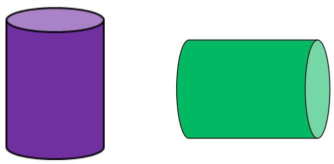
When you cut a piece off a shape,
you reduce the original shape's area and perimeter.

If you multiply a whole number by a fraction between 0 and 1,
the product will be greater than the whole number.

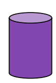
Write your own:

Thoughts about a Cylinder

- What are a few real-life examples of cylinders?
- What vocabulary words can be used to describe a cylinder?



Volume of a Cylinder



Using two sheets of 8.5 by 11 inches of paper, create two cylinders. One with a height of 11 inches and one with a height of 8.5 inches. How do the volumes of the cylinders compare?

Prediction: ______________________

Work to support or refute your prediction:

---Fisher, P & Blanchowicz, C. *A Few Words about Math and Science*, ASCD November 2013, p. 48

From Problem Performers to Problem Solvers

Problem Performers	Problem solvers
Do it just like the teacher did.	Think before you start.
Write your numbers straight.	Draw a picture to understand the problem.
Don't talk when you work. That's cheating.	Explain how you solved it, using math vocabulary.
Find an answer and move on.	Check your work. Do it a different way.

3 Questions

Mathematics can be used to quantify the world. After each picture is shown, generate mathematical questions you first think of and then identify what you would need to know to answer each question. This helps students when problem solving answer: What do I know? What do I need to know?

Picture 1

Question	What do you need to know?

Problem Solving:

Picture 2

Question	What do you need to know?

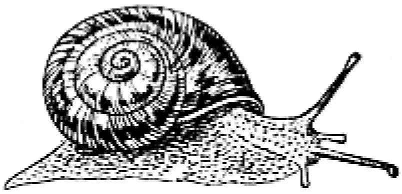
Problem Solving:

The Task Analysis Guide

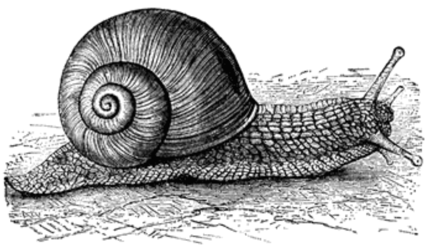
(**Source:** This table is provided verbatim from Smith & Stein, "Selecting and Creating Mathematical Tasks: From Research to Practice," *Mathematics Teaching in the Middle School*, February 1998, *3*(5), p. 348)

Lower Level	Higher Level
Memorization Task • Involve reproducing previously learned facts, rules, formulae, or definitions to memory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use procedure. • Are not ambiguous—such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.	**Procedures With Connections Tasks** • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections Tasks • Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. • Require limited cognition demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedure being used. • Are focused on producing correct answers rather than developing mathematical understanding. • Require no explanations, or explanations that focus solely on describing the procedure that was used.	**Doing Mathematics Tasks** • Require complex and no algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or worked-out example). • Require students to explore and understand the nature of mathematical concepts, processes, or relationships. • Demand self-monitoring or self-regulation of one's own cognitive processes. • Require students to access relevant knowledge and experiences and make appropriate use of them in working through her task. • Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • Require considerable cognitive effort and may involve some level of anxiety for the student due to unpredictable nature of the solution process required.

The Great Snail Race

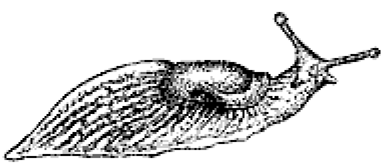


Snail A takes 20 minutes to travel 3 inches



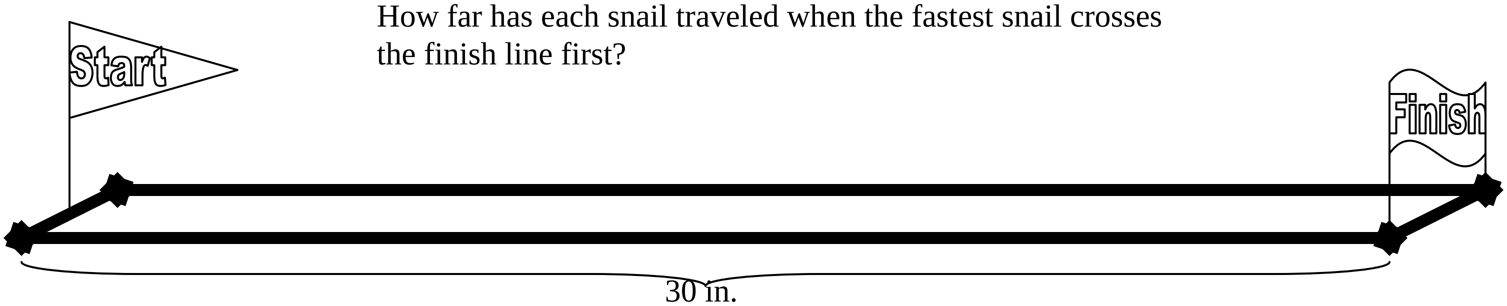
Snail B travels 5 inches in 10 minutes

Snail C travels 1 inch every 15 minutes.



Snail D takes 30 minutes to travel 6 inches

How far has each snail traveled when the fastest snail crosses the finish line first?



Adapted from Daro, Phil (2011, January). Mathematics common core state standards. Mathematics Educator Appreciation Day Key Note Address.

HS Task
A-REI, A-CED

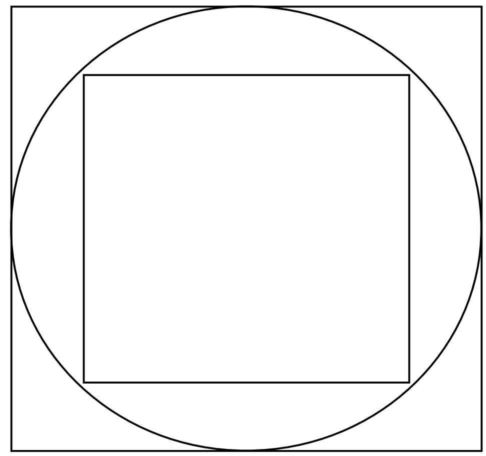
Nola was selling tickets at the high school dance. At the end of the evening, she picked up the cash box and noticed a dollar lying on the floor next to it.

She said, “I wonder whether the dollar belongs inside the cash box or not.”

The price of tickets for the dance was 1 ticket for $5 (for individuals) or 2 tickets for $8 (for couples). She looked inside the cash box and found $200 and ticket stubs for the 47 students in attendance. Does the dollar belong inside the cash box or not?

Circles and Squares

This diagram shows a circle with one square inside and one square outside.



1. What is the ratio of the areas of the two squares?
   Show your work

   ____

2. If a second circle is inscribed inside the smaller square, what is the ratio of the areas of the two circles? Explain your reasoning.

   ____

Shifts in Classroom Practice

Shift 1: From *same instruction* toward *differentiated instruction*.

Same instruction for all students.	Differentiated instruction but same learning outcomes for all students.

Shift 2: From *students working individually* toward *community of learners.*

Students work individually on tasks and seek feedback from teacher on reasonableness of strategies and solutions.	Community of learners where students hear, share, and judge reasonableness of strategies and solutions.

Shift 3: From *mathematical authority coming from the teacher or textbook* toward *mathematical authority coming from sound student reasoning.*

Correctness of solutions is determined by seeking input from teacher or textbook.	Correctness of solution is based on reasoning about the accuracy of the solution strategy.

Shift 4: From *teacher demonstrating 'how to'* toward *teacher communicating 'expectations' for learning.*

Teacher demonstrates the way in which to solve a problem and helps students in solving the problem in that way.	Teacher facilitates high-level performance by sharing learning goals and expectations for products that demonstrate learning.

Shift 5: From *content taught in isolation* toward *content connected to prior knowledge.*

Content presented independent of its connections to what has been previously learned.	Content presented in ways where explicit attention is given to making connections among mathematical ideas.

Shift 6: From *focus on correct answer* toward *focus on explanation and understanding.*

Discussions and classroom routines focus on student explanation of how they solved a task and if it is correct.	Discussions and classroom routines focus on student explanations that address why an answer is (or isn't) correct.

Shift 7: From *mathematics-made-easy for students* toward *engaging students in productive struggle.*

Mathematics is presented in small chunks and help is provided so that students reach solutions quickly and without higher level thinking.	Teacher poses tasks and challenges students to persevere and attempt multiple approaches to solving problems.

Used with permission. From ***"Coaching Tools for Supporting the Common Core State Standards for Mathematical Practice,"*** presented at the NCSM Annual Conference Philadelphia, PA, Monday, April 23, 2012, by Maggie B. McGatha and Jennifer M. Bay-Williams.

Shifts in Classroom Practice

Choose one shift in classroom practice for you and your team to commit to implementing as soon as next week. Complete the Frayer model together to gain a collective understanding of the shift.

What does it look like?	What does it sound like?
Examples	Nonexamples

Figure 2.21

Mathematical Lesson-Planning Tool

Unit: **Date:** **Lesson:**
Essential learning standard: List the essential learning standard for the unit addressed by today's lesson.
Learning objective: As a result of class today, students will be able to . . .
Essential Standard for Mathematical Practice: As a result of class today, students will be able to demonstrate greater proficiency in which Standard for Mathematical Practice?
Formative assessment process: How will students be expected to demonstrate mastery of the learning objective during in-class checks for understanding, teacher feedback, and student action on that feedback?

Probing Questions for Differentiation on Mathematical Tasks	
Assessing Questions (Create questions to scaffold instruction for students who are "stuck" during the lesson or the lesson tasks.)	**Advancing Questions** (Create questions to further learning for students who are ready to advance beyond the learning standard.)

Tasks (Tasks can vary from lesson to lesson.)	**What Will the Teacher Be Doing?** (How will the teacher present and then monitor student response to the task?)	**What Will the Students Be Doing?** (How will students be actively engaged in each part of the lesson?)
Beginning-of-Class Routines How does the warm-up activity connect to students' prior knowledge, or how is it based on analysis of homework?		

Task 1 How will the students be engaged in understanding the learning objective?		
Task 2 How will the task develop student sense making and reasoning?		
Task 3 How will the task require student conjectures and communication?		
Closure How will student questions and reflections be elicited in the summary of the lesson? How will students' understanding of the learning objective be determined?		

Source: Kanold, T. D. (Ed.), Kanold, T. D., & Larson, M. (2012). Common Core mathematics in a PLC at work, leader's guide. *Bloomington, IN: Solution Tree Press. (See pp. 53–54.)*

Effective Classroom Communication

- How do students express their ideas, questions, insights, and difficulties?
- Where are the most significant conversations taking place (student to teacher, student to student, teacher to student)?

Do students see each other as reliable and valuable resources?

Lesson Design

- What standard(s) are *students learning*?
- How will formative assessment processes be built into the plan?
- What is your plan for students who will get stuck?
- What is your plan for students needing enrichment?
- Which practice are you developing in students as a habit of mind?
- Which tasks are you using in your lesson? What is the teacher doing? What are students doing?
- How will your lesson begin and end?

Reflection

- What are **2** ways you can include the process standards in lesson design?
- What is **1** way you can assess student learning during a lesson?